JEE (Main)-2018
Paper-II

1. If f(x) + 2f(1 – x) = x^2 + 1, $\forall x \in R$, then the range of f is:

(a) $\left(-\infty, \frac{1}{3}\right]$ (b) $\left[-\frac{1}{3}, \infty\right)$

(c) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (d) $\left[\frac{1}{3}, \infty\right)$

2. Let $A\{z \in C : |z| = 25\}$ and $B\{z \in C : |z + 5 + 12i| = 4\}$. Then the minimum value of $|z - \omega|$, for $z \in A$ and $\omega \in B$, is:

(a) 6 (b) 7 (c) 8 (d) 9

3. If the product of the roots of the equation $x^2 - 5kx + 2e^{2\log_e|k|} - 1 = 0$ is 49, then the sum of the squares of the roots of the equation is:

(a) 525 (b) 527 (c) 576 (d) 627

4. If $A = \begin{bmatrix} 2 & 52 & 152 \\ 4 & 106 & 358 \\ 6 & 162 & 620 \end{bmatrix}$, then the determinant of the matrix adj (2A) is equal to:

(a) 64 (b) 256 (c) 2048 (d) 4096

5. Let S be the set of all real values of λ for which the system of linear equations

$$\lambda x + y + z = 5\lambda$$
$$2\lambda x + 2y - z = 1$$
$$3y + z = 9$$

has infinitely many solutions. Then S;

(a) equals R

(b) is a singleton

(c) contains exactly two elements

(d) is an empty set

6. In order to get through in an examination of nine papers a candidate has to pass in more papers than the number of papers in which he fails. The number of ways in which he can fail, in this examination is:

(a) 128 (b) 255

(c) 256 (d) $9 \times (8)!$

7. Let T_r denote the r^{th} term in the binomial expansion of $(a + 1)^{50}$. If $T_{25} + T_{27} = \frac{125}{52} T_{26}$ then the sum of all the values of a is:

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

8. In an ordered set of four numbers, the first 3 are in A.P. and the last 3 are in G.P. whose common ratio is 7/4. If the product of the first and fourth of these number is 49, then the product of the second and third of these is:

(a) 60 (b) 112 (c) 128 (d) 144

9. If $e^{(\sin^2 x + \sin^4 x + \sin^6 x + ... + ad\ inf.)\log_e 2}$ $\left(0 < x < \frac{\pi}{2}\right)$ satisfies the equation $y^2 - 5y + 4 = 0$, then $\frac{\sin x}{\cos x - \sin x}$ is equal to:

(a) $-\left(2 + \sqrt{2}\right)$ (b) $-\left(\sqrt{2} + 1\right)$

(c) $\sqrt{2} - 1$ (d) $2 + \sqrt{2}$

10. Let f(x) = $x\left[\frac{1}{x}\right]$ for all $x (\neq 0) \in R$ where for each $t \in R$, $[t]$ denotes the greatest integer less than or equal to t. Then:

(a) $\lim_{x \to 0+} f(x) = 0$ (b) $\lim_{x \to \frac{1}{3}+} f(x) = 1$

(c) $\lim_{x \to \frac{1}{2}-} f(x) = 1$ (d) $\lim_{x \to 2-} f(x) = 1$

11. If $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & \\ k\sqrt{2}\log_e 2\log_e 3, & x=0\end{cases}$ is a continuous function in the interval $[0, 2\pi)$, then k is equal to:

(a) 4 (b) 18 (c) 24 (d) 36

12. If y = y(x) is an implicit function of x given by ycosx + xcosy = π; then y"(0) is equal to:

(a) π (b) $-\pi$ (c) 0 (d) 2π

13. For each $x \in R$, let f(x) = |x – 1|, |x – 1|, g(x) = cos x and $\phi(x) = f(g(2\sin x) - g(f(x))$.

Then ϕ is:

(a) differentiable at each point of R

(b) not differentiable at 0

(c) not differentiable at 1

(d) differentiable only in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

14. If f(x) = $|x^2 - 16|$ for all $x \in R$, then the total number of points of R at which f : R →R attains local extreme values is:

(a) 1 (b) 2 (c) 3 (d) 4

15. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}dx, \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx,$ then J – i equals:

(a) $\frac{1}{2}\log_e\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right|+C$

(b) $\frac{1}{2}\log_e\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right|+C$

(c) $\frac{1}{2}\log_e\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right|+C$

(d) $\frac{1}{2}\log_e\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}-e^{2x}+1}\right|+C$

16. If $\int x^5\sqrt{\frac{1+x^2}{1-x^2}}\,dx = m\pi + n$, then the ordered pair (m, n) is equal to

(a) $\left(\frac{1}{3}, \frac{1}{8}\right)$ (b) $\left(\frac{1}{8}, \frac{2}{3}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (d) $\left(\frac{1}{8}, \frac{1}{3}\right)$

17. The area (in sq.units) of the region bounded by the curve, 12y = 36 – x^2 and the tangent drawn to it at the points, where the curve intersects the x-axis is

(a) 12 (b) 18 (c) 27 (d) 6

18. Let y = y(x) be the solution of the differential equation: $x\log e^x\frac{dy}{dx}+y=3x\log_e x, (x>1)$. If y(e) = 0, then y($e^2$) is equal to

(a) e^2 (b) $\frac{1}{2}e^2$ (c) $\frac{3}{2}e^2$ (d) $3e^2$

19. Let the straight lines, 5x – 3y + 15 = 0 and 5x + 3y – 15 form a triangle with the x-axis. Then the radius of the circle circumscribing this triangle is

(a) $\frac{8}{5}$ (b) $\frac{17}{5}$ (c) $\frac{12}{5}$ (d) $\frac{16}{5}$

20. The mirror image of the circle $x^2 + y^2 - 10x - 10y = 0$ in the line x + y + 5 = 0 is a circle passing through the point:

(a) (-3, -7) (b) (-9, -7)

(c) (-3, -11) (d) (-9, -11)

21. Let S the focus of the parabola, $x^2 + 8y = 0$ and Q be any point on it. If P divides the line segment SQ in the ratio 1:2, then the locus of P is

(a) $9x^2 + 24y + 32 = 0$
(b) $9y^2 + 32 = 0$
(c) $32x^2 + 24x + 32 = 0$
(d) $32y^2 + 27x + 36 = 0$

22. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the eccentricity of the hyperbola $x^2 - \cos^2\theta - y^2 = 6\cos^2\theta$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 + y^2\cos^2\theta$ then θ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\frac{\pi}{3}$

23. If the line $\frac{x-1}{4} = \frac{y+3}{2} = \frac{z+5}{1}$ lies in the plane $2x + ly + mz = 16$, then $l^2 + m^2$ is equal to

(a) 16 (b) 20 (c) 98 (d) 85

24. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$ and $\hat{r}\left(\hat{i} - \hat{j}\right) + 4 = 0$ and perpendicular to the plane $\hat{r}\left(2\hat{i} - \hat{j} - \hat{k}\right) + 4 = 0$, is:

(a) $\vec{r}.\left(\hat{i} - 2\hat{j} + 4\hat{k}\right) = 3$
(b) $\vec{r}.\left(\hat{i} - 2\hat{j} + 4\hat{k}\right) = 5$
(c) $\vec{r}.\left(2\hat{i} - \hat{j} + 5\hat{k}\right) = 3$
(d) $\vec{r}.\left(2\hat{i} - \hat{j} + 5\hat{k}\right) = 5$

25. If $\hat{a}, \hat{b}, \hat{c}$ be three unit vectors, $\hat{b}$ *and* $\hat{c}$ are non-parallel, such that $\hat{a} \times \left(\hat{b}, \hat{c}\right) = \frac{\hat{b} + \hat{c}}{2}$, then the angle between $\hat{a}$ *and* $\hat{b}$ is:

(a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

26. A box contains 6 red ball and 2 black balls. Two balls are drawn, at random, from it without replacement. If X denotes the number of red balls drawn then E(X) is equal to:

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{5}{2}$ (d) $\frac{27}{28}$

27. A six faced die is so biased that it is thrice likely to show an even number than an odd number, when thrown. If the die is thrown twice, the probability that sum of the numbers on the die is even is:

(a) $\frac{3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{7}{9}$ (d) $\frac{3}{8}$

28. The total number of $x \in [0, 2\pi]$ which satisfy the equation $4(\cos^{10}x + \sin^2 x) = 4 + \sin^6 x \sin^2(2x)$, is:

(a) 2 (b) 3 (c) 5 (d) 6

29. $\tan\left(\frac{1}{2}\sin^{-1}\frac{4}{5} + \frac{1}{2}\cos^{-1}\frac{15}{17}\right)$ is equal to:

(a) $\frac{6}{7}$ (b) $\frac{2}{3}$ (c) $\frac{4}{15}$ (d) $\frac{9}{22}$

30. The Boolean expression $(p \wedge q) \vee ((\sim q) \vee p)$ is equivalent to:

(a) $\sim p \vee q$ (b) $\sim q \vee p$
(c) $p \vee q$ (d) $(\sim p) \vee (\sim q)$